FINANCIAL MARKET RISK ANALYSIS THROUGH CROSS-CORRELATION’S EIGENVECTOR COMPONENTS DISTRIBUTION

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Objective

- Random matrix theory (a statistical physics method) application to the cross-correlation matrix of the stock market
- Study of eigenvalue spectrum and eigenvector can reveal useful information about the inter-dependency of the stocks
- Market risk – the un-diversifiable risk can be determined through this analysis
Outline

1. Introduction
2. Empirical results
3. One-factor model of correlation matrix
4. Market risk and the comp. of the 1st eigenvector
5. Future work and Conclusion
Introduction

- Stock price & daily price change
  \[ P_t, r_t = \ln \left( \frac{P_t}{P_{t-1}} \right) \]

- Randomness
  \[ r_t = \mu \Delta t + \sigma \times \varepsilon_t \]

- The stock market: An inter-dependent system of hundreds to thousand stocks
Introduction

- **Portfolio**
  
  \[(\omega^1, \omega^2, \ldots, \omega^N)\]

- **Portfolio risk**

  \[Risk_p = \sum_i (\omega^i)^2 \cdot \text{var}(r^i) + \sum_{i < j} \omega^i \cdot \omega^j \cdot \text{cov}(r^i, r^j)\]

- **Diversification**

  \[Risk_p = \text{Market} \_ \text{risk} + \text{Diversifiable} \_ \text{risk}\]
Introduction

- Portfolio
  \((\omega^1, \omega^2, \ldots, \omega^N)\)

- Portfolio risk
  
  \[ \text{Risk}_p = \sum_i (\omega^i)^2 \cdot \text{var}(r^i) + \sum_{i<j} \omega^i \cdot \omega^j \cdot \text{cov}(r^i, r^j) \]

- Diversification
  
  \[ \text{Risk}_p = \text{Market\_risk} + \text{Diversifiable\_risk} \]
Market risk

- Market risk (systematic risk) is a notion of the (complex) system, not of individual stock
- Market risk depends on the relation/correlation between stocks (components in the system)
- No correlation = no market risk = no risk at all (as $N \to \infty$)
- Stock price is often embedded by noise
- Random matrix method is therefore a suitable tool
Random Matrix Theory (RMT)

- Random entries
- Eigenvalue spectrum

\[ Q = \frac{T}{N} \rightarrow \gamma \text{ (finite)} \]

The limiting spectra is given by \textit{Marcenko-Pastur Law}

\[ g(\lambda) = \frac{Q}{2\pi} \frac{\sqrt{(\lambda_+ - \lambda)(\lambda - \lambda_-)}}{\lambda} , \]

\[ \lambda_- \leq \lambda \leq \lambda_+, \lambda_+ = 1 + 2 \sqrt{\frac{1}{Q} + \frac{1}{Q}} \]

\[ C_{\text{RMT}} = \begin{pmatrix} 1 & \varepsilon_{1,2} & \cdots & \varepsilon_{1,N} \\ \varepsilon_{2,1} & 1 & \cdots & \varepsilon_{2,N} \\ \vdots & \vdots & \ddots & \vdots \\ \varepsilon_{N,1} & \varepsilon_{N,2} & \cdots & 1 \end{pmatrix} \]

\[ \varepsilon \sim \text{i.i.d.} N(0,1) \]

\[ C_{\text{RMT}} \mathbf{u}^i = \lambda^i \mathbf{u}^i \]
Empirical data

- $\lambda_1$: market mode
- $\lambda_{>\lambda_+}$: sector modes
- $\lambda_{<\lambda_+}$: noise

Empirical data

- Vietnamese market

Q. Nguyen, Physica A 2013
Largest eigenvalue

- Different group reported different value of $\lambda_1$ in the range of 20-200, about 10-50 $\lambda_+$
- This mode is the collective behaviour of all stock represent by their correlation, which is not null

- The value of $\lambda_1$ must be derived from non-null value of collective correlation
Largest eigenvalue

- We found a phenomenological relation

\[ \lambda_1 \sim N \times < \rho > \]

(an analytic derivation of this formula is expected in the future work)

- How can this average correlation explain the eigenvalue spectra?
One-factor model

Constant correlation model

\[ C = \begin{pmatrix} 1 & \rho_0 & \ldots & \rho_0 \\ \rho_0 & 1 & \ldots & \rho_0 \\ \vdots & \vdots & \ddots & \vdots \\ \rho_0 & \rho_0 & \ldots & 1 \end{pmatrix} \]

\[ r^t \sim N(0, C) \]
Empirical data - IPR

\[ I^k = \sum_{l=1}^{N} [u_l^k]^4 \]

Empirical data

Simulation data

(b) \[10^0 \]

Inverse participation ratio

\[ 10^{-1} \]

\[ 10^{-2} \]

\[ 10^0 \]

\[ 10^1 \]

\[ 10^2 \]

Eigenvalue \( \lambda \)

Random IPR

Random eigenvalue

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Eigenvectors

- Eigenvector 1
- Eigenvector 2
- Eigenvector 45
- Eigenvector 90
1st eigenvector

- Positive
- Relatively uniform
- Some groups found relation to the stock market capital - We do not find such dependence
1st eigenvector

However, it is natural to think about the correlation of each stock to all others stocks.

Influence factor:

\[ IF_i = \sum_j \rho_{ij} \]
1\textsuperscript{st} eigenvector

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Influence factor:

\[ IF_i = \sum_j \rho_{ij} \]

Q. Nguyen et al., in preparation
1st eigenvector

- Volume

![Graph showing the relationship between Component of $U^{\text{max}}$ and Average daily volume.](image)
1\textsuperscript{st} eigenvector

- Minimum Spanning Tree and U\textsuperscript{1}'s components
1st eigenvector’s components

- The distribution of IF is not random
- High $u^1_i$ = high IF: stock that correlate the most to all other stocks

<table>
<thead>
<tr>
<th>DEVELOPED MARKET</th>
<th>EMERGING MARKET</th>
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<tr>
<td>- Institutional investors</td>
<td>- Retail investors</td>
</tr>
<tr>
<td>- Fundamental driven</td>
<td>- Herding driven</td>
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<tr>
<td>- Market leading: high market cap. stock</td>
<td>- Market leading: popular stock</td>
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<td></td>
<td>(no def.)</td>
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<td></td>
<td>(the case of Vietnam: securities stocks)</td>
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U^1 portfolio

- Is the highest coherent mode
- Is not the commonly defined market portfolio (derived from the Efficient Market Theory)
- Its risk is higher than the EMT’s
- It has high market risk but still eliminate specific risk
Nature of stock correlation

Why stock correlation is almost positive?
Why stock corr. is almost positive?

- Firm $i$:

$$O^i = F^i (S^i, M)$$

$$\frac{\partial F^i}{\partial M} > 0$$

$$O^j_t = F^i (S^i, \sum_j O^j_{t-1})$$

$$O^j_t = F^i (S^i, O^1_{t-1} + O^2_{t-1} + \ldots + O^N_{t-1})$$

Therefore, $\rho_{ij}$ is very likely to be positive unless there is a strict negative correlation between $S^i$ and $S^j$. 
Source of instability?

The market:

\[
\begin{align*}
O_t^1 &= F^1 (S^1, O_{t-1}^1 + O_{t-1}^2 + \ldots + O_{t-1}^N) \\
O_t^2 &= F^2 (S^2, O_{t-1}^1 + O_{t-1}^2 + \ldots + O_{t-1}^N) \\
& \quad \ldots \\
O_t^N &= F^N (S^N, O_{t-1}^1 + O_{t-1}^2 + \ldots + O_{t-1}^N)
\end{align*}
\]

- Positive feedback
- Non-linearity

Agent-based simulation could be the efficient tool
Conclusions

- One factor model:
  - Magnitude of the maximum eigenvalue
  - Shift of the bulk

- 1st eigenvector analysis
  - 1st eigenvector’s component ~ Influence factor
  - Discuss the market mode
Selected References

- Q. Nguyen, D. Nguyen, T. Hoang, P. Huynh, *in preparation*
THANK YOU FOR YOUR ATTENTION!